Unorthodox abstract models for the OFDMA multihop transmission

Rainer Schoenen, ComNets, FB6, RWTH Aachen

***Abstract*— Wireless systems are complex devices and a lot of research is done to improve parts of these systems and get a better understanding of the total behaviour. One step towards an easier understanding is the ISO-OSI reference model which decomposes the system into layers. But still for each layer, easy models are missing where you can learn the behaviour within a few minutes. Researchers like to use abstract models in order to support an analytic approach and prefer detailed models that can only be evaluated by means of simulation. If this abstract view is desired, or a different view on a problem might be useful, an unconventional model can be handy. This paper introduces two models, an electric equivalent circuit and a Petri Net model, that were known for many decades from other technical areas and is used here to describe aspects of cellular radio systems, especially based on OFDM. They are not meant to be more accurate, but instead rather more to round off the high-level understanding. The models are used to show the impact of multihop techniques, i.e. store-and-forward relays in cellular radio networks, which is quite an important topic for the cost efficient provision of coverage and capacity.**

I. Introduction

MODELLING of electrotechnical systems is always a tightrope walk between accuracy and clearness. Researchers like to use abstract models in order to support an analytic approach and prefer detailed models that can only be evaluated by means of simulation. In radio systems engineering, the mathematical models on the physical layer (PHY) are mature and complex models and formulas are very common. Sometimes it is desired to impart an overview knowledge about a topic or to get a different view on a problem from a standpoint that insiders cannot see anymore. That is when unconventional models come into place. They are not meant to be more accurate, but instead rather more to round off the high-level understanding.

In this paper an electric equivalent circuit is introduced which models the bit stream betweeen base stations (BS) and user terminals (UT) as well as the required OFDMA resources depending on the possible PhyModes, which in turn depend on the pathloss, i.e. the distance between BS and UT. Especially when using multihop techniques, i.e. store-and-forward relays in cellular radio networks, the model can be used to learn the benefit of relaying. It can also be used for quantitative calculations, e.g. to determine the suitable resource partitioning scheme between first and second hop, or to find out the fractions of greedy (best effort) data streams that UTs will get if the resource scheduler assigns resources fairly among UTs.

These multihop radio systems are very beneficial due to their improvements of coverage and capacity. For that reason they they are proposed for the next generation cellular systems like 3G-LTE [1]. The system capacity or spectral efficiency $[bits/s/Hz]$ is improved without the need of more BSs with fiber access. In regular hexagonal scenarios [2] and city scenarios [3] this has been shown. Figure 1 shows the cellular layouts for typical multihop-augmented cells.

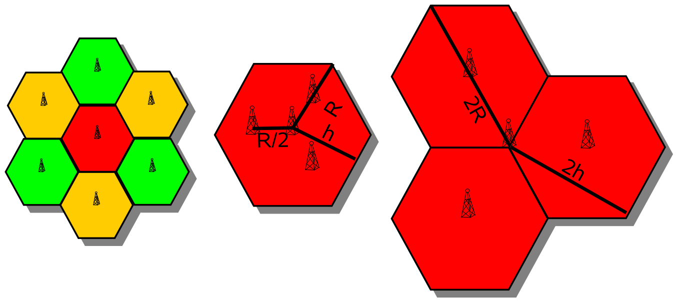

Fig. 1. Left: Conventional cellular geometry, Middle: Geometry to increase capacity, Right: Geometry to increase coverage

The data rate of the raw data streams on the physical layer (PHY) can be determined with the first model. On the data link layer (DLL) there must be a method to deal with the packet errors (after FEC) that occur due to the nature of the wireless channel. Typically the logical link control provides an automatic repeat request (ARQ) instance for each data flow so that errored or missing packets are retransmitted. In this paper a Petri-Net (PN) is shown that models the logical behaviour by its structure as well as the timing and performance by using stochastic PN models.

The paper is organized as follows. The first section discusses the electric equivalent model and its application to resource partitioning. The second section explains the PN model and some results from it. The paper ends with a concluding summary.

II. Modelling OFDMA Resources

OFDMA Resources on the radio link are the scarce resource that all UTs compete for when the offered traffic load is greedy, i.e. there is more traffic than capacity. This is the assumed situation for best effort traffic and typical file transfer or peer-to-peer applications.

A. OFDMA Resources

OFDM has evolved as an efficient multiplex scheme of typically 1024 small orthogonal subcarriers within the system bandwidth. The big advantage of OFDM is that different modulation and coding schemes (MCS,PhyMode) are possible for each subcarrier, so a robust BPSK can be used on frequencies where the channel is currently bad due to fading, while e.g.

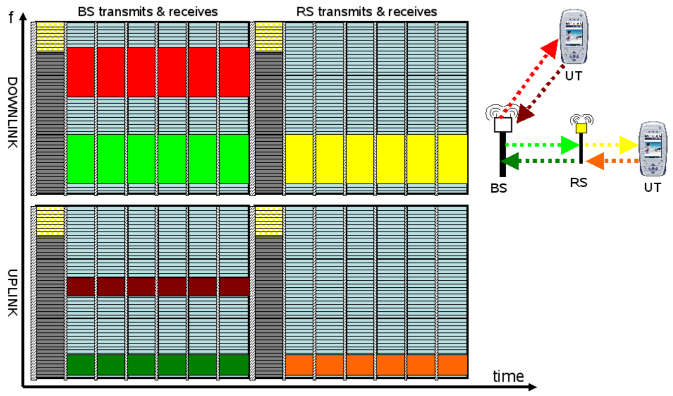

Fig. 2. OFDMA resources in a multihop scenario

QAM64 can be used on more stable subcarriers with higher $SINR$ values. This is called dynamic adaptive modulation and coding (AMC) [4]. OFDM**A** means that within one OFDM symbol, several different UT receivers can be addressed. So for example, 256 subcarriers are used for each of four UTs that receive the symbol. Dynamic subcarrier assignment (DSA) selects the best resources for each UT based on channel state information (CSI). As the distance and pathloss may be very different among the UTs, it is likely that the PhyModes used for each UT are very different within the full OFDM symbol.

Close to the base station, the higher received $SINR$ value allows the highest Modulation&Coding scheme (PhyMode $64QAM - 5/6$ for LTE), which offers the highest raw data rate of approximately $84Mbit/s$. At the cell border and in significant fractions of the cell area, the offered data rate is one order of magnitude lower. What makes this even worse is that some terminals operating at the lowest PhyMode occupy a ten times bigger part of the spectrum than the same number of terminals operating at the highest PhyMode. That means the average cell capacity is overproportionally determined by the maximum possible rate at the outer regions.

The organization of resources is in time and frequency, see Figure 2. Subcarriers are grouped together to form sub-channels, within which all subcarriers are treated the same. For a broadband FDD LTE system there are 100 subchannels available. Within a radio cell the resources are unique and non-overlapping to avoid intra-cell interference completely. The BS coordinates the resources such that each link uses its own subset of the resources. Especially the second hop, within which the resources are scheduled under control of the relay node (RN), must be assigned a resource subset, so that it operates independent of the first hop. This is called resource partitioning and is an important part of the relaying organization. Without proper control the fairness of all UTs is not achieved [5] whereas the fairness goal should be to offer each UT the same amount of resources [6]. In this case, UTs with bad PhyMode get less share of the data rate. On the other hand, this maximizes the total capacity under this fairness contraint.

B. The Electric Equivalent Circuit

In Figure 3 an Electrical Circuit Equivalent is shown which can describe the distribution of data rates to greedy user terminals (UT) in a radio cell, especially when there are relay nodes (RN).

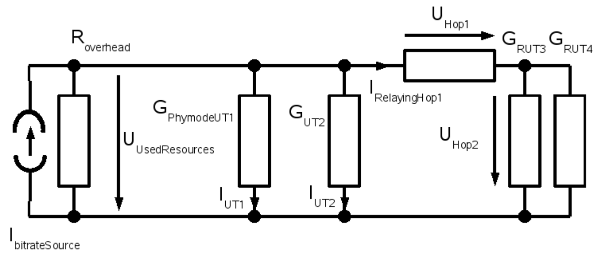

Fig. 3. Electrical Circuit Equivalent of a Relay Cell

The equivalents are given in Table I. In this equivalent model, an electrical current has the meaning of a data stream, and the strength of the current means the data rate in $[bits/s]$ through the block diagram element. The current source to the left is the base station (BS) and its downlink (DL) traffic. At each junction the traffic splits, and Kirchhoff's Current Law ensures the consistency (no loss). In this model, a voltage means the amount of OFDMA resources available or used. A voltage source means a BS which has a certain number of OFDM subcarriers available, which gives a "voltage" in $Symbols/s$.

E.g. for 3GPP-LTE, this is $1200 \cdot 7Symbols/500\mu s = 16800000Symbols/s$ raw (PHY) data rate. As it is the physical meaning of voltage, this is in a similar way the "potential" of the transmission, something proportional to the system bandwidth. According to Norton and Thevenin theorems, a voltage source and current source together with its internal resistance can always be transformed into each other. Therefore only a current source is drawn in Figure 3 instead of a voltage source. The resistors shown have a characteristic conductance G, which is proportional to the PhyMode capability, i.e. they can conduct a number of bits per symbol, e.g. 5 bits for $QAM64 - 5/6$.

This is exactly the mutual information (MI) that can be calculated for a given $SINR$ and PhyMode [7]. Of course this G depends on the distance (due to pathloss and received $SINR$) at the UT. Ohm's Law $I = U \cdot G$ can be satisfied with these equivalent definitions. All user terminals UT_x are greedy here, i.e. they try to get as much datarate as possible. Let the distribution of resources be fair such that each UT gets an equal share of the OFDM resources. A user terminal

item	original unit/def.	equivalent	eq. unit
voltage source	[U]=V=W/A	BaseStation	Symbols/s
voltage	[U]=V=W/A	UsedOFDMResources	Symbols/s
current source	[I]=A	BaseStation	Bits/s
current	[I]=A	DataRate	Bits/s
conductance	[G]=V/A	PhyMode	Bits/Symbol
Ohm's Law	G=I/U	ResourceUsageLaw	Bits/Symbol

TABLE I

EQUIVALENT MEANING FOR THE ELECTRIC CIRCUIT MULTIHOP SYSTEM

UT_x attached to the base station then receives a datarate proportional to its conductance G by construction, because Kirchhoff's Current and Voltage Laws hold.

If there is a relay in the circuit as shown on the right, there are resources needed on the first hop (U_{Hop1}), which are "wasted", and the resources used on the second hop (U_{Hop2}) are shared while the datarate splits to one of the remote user terminals RUT_x. Despite the waste in the first hop, due to improved Phymodes (RUTs and RN are close together) there can be a higher datarate through RUT_x in the multihop case compared to a UT in a singlehop case where the Phymode is bad (low G).

Figure 4 shows an example for greedy traffic. Let $G_{UT3} = G_{UT4} = 5Bits/Symbol = 5bps$ because they are close to the RN. Let $G_{RN,Hop1}$ also be $2 \cdot 5bps$ because there is a high antenna gain between BS and RN and the factor 2 is needed for two supported RUTs. Then the parallel resistors can be combined to give $G_{Hop2} = 10bps$. The remaining two resistors in series are equivalent to one with $G_R = 5bps$. This means they compete with a singlehop UT of the same capability $G_{UT} = 5bps$. Both will get the same amount of resources altogether. But for the two-hop transmissions, half of the resources are given to the first hop, half to the second hop. So $U_{Hop2} = U_{Source}/2$ and the resulting data rates I_{RUTx} are half of that of a comparable singlehop UT. In this model, the currents I_{UTx} are proportional to the data rate the UTs would get, so they provide useful relative results under the condition that each flow must get an equal share of the resources. For absolute results, the "voltage" source must be scaled (divided) by the number of participants of the resources.

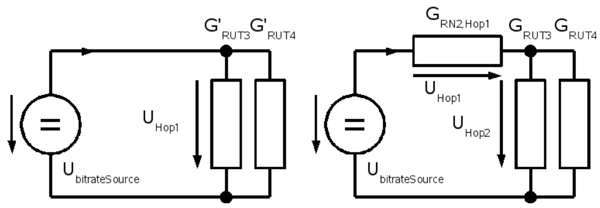

Fig. 4. Left: Singlehop UTs; Right: 2-hop RUTs; The PhyModes are different.

For the singlehop UTs, according to Figure 3, the data rate they get is $I_{UT1} = U_{Resources} \cdot G_{UT1}$ and $I_{UT2} = U_{Resources} \cdot G_{UT2}$. For these four UTs, $U_{Resources}$ is $16800000 Symbols/s/4 = 4200000 Symbols/s$. Let's assume G_{UT1} is $5bps$ ($QAM64 - \frac{5}{6}$) and G_{UT2} is $2/3bps$ ($QPSK - \frac{1}{3}$). Then the data rates are $I_{UT1} = 21Mbps$, $I_{UT2} = 2.8Mbps$, $I_{RUT3} = I_{RUT4} = 10.5Mbps$ and the OFDMA channel is fully utilized with that. The data rates (as the current does) add up to a total of $I_{tot} = 44.8Mbps$.

In general the resource partitioning and data rates can be calvulated like that. In the special case when all UTs have the same PhyMode and there are as many UTs ans RUTs, the suitable resulting resource partitioning is shown in Figure 5 [6]. It is interesting to note that the links $BS \rightarrow UT$, $BS \rightarrow RN$, $RN \rightarrow RUT$ all get $1/3$ of the total resources (the $BS1$ resource block is used for $BS \rightarrow UT$ and $BS \rightarrow RN$).

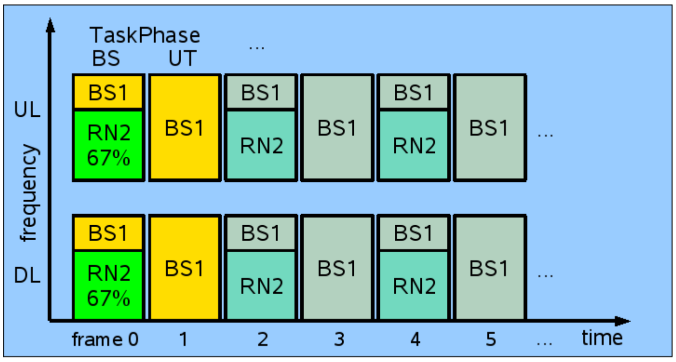

Fig. 5. Resource partitioning between BS and RN. Shown here is which BS or RN uses which resource subset.

C. Fair Share Partitioning

We can now derive the calculations for fair share partitioning and scheduling with arbitrary PhyModes [6]. The "proportional fair share" means that any UT gets the same amount of resources R_i on its last hop. This amount of resources leads to a higher data rate r_i for those UTs i using a higher PhyMode. Let MCS_i be the PhyMode on the last hop for UT_i or RUT_i and MI_i be the mutual information in $[bits/s/Hz]$ for this PhyMode [7]. MI_{RN} be the mutual information of $hop1$ between BS and RN of a multihop connection. MI_{max} is $5bit/s/Hz$ for $QAM64 - 5/6$. The amount of resources needed is $R_i = c \cdot \frac{r_i}{MI_i}$ with a constant c that is cancelled out in the equations, later. The resources are limited as

$$R_{tot} = c\frac{r_{tot}}{MI_{max}} = \sum R_i = \tag{1}$$

$$c\sum_{i=1}^{\#UTs}\frac{r_{UT_i}}{MI_{UT_i}} + c\sum_{i=1}^{\#RUTs}\frac{r_{RUT_i}}{MI_{RUT_i}} + c\frac{\sum_{i=1}^{\#RUTs} r_{RUT_i}}{MI_{RN}}$$

Eq. (1) can be used to calculate the data rates of each UT and the required resources. If the fairness policy is to grant each UT the same amount of resources R_i and to each RUT twice this number[1] (to support $hop1$ and $hop2$ and a fair competition between UTs and RUTs) then we get using $c = \frac{R_{tot} \cdot MI_{max}}{r_{tot}}$

$$R_i = \frac{R_{tot}}{\#UTs + 2 \cdot \#RUTs} \tag{2}$$

$$r_i = \frac{R_i \cdot M_i}{c} = \frac{r_{tot}}{\#UTs + 2 \cdot \#RUTs} \cdot \frac{MI_i}{MI_{max}} \tag{3}$$

r_{tot} is the total maximum data rate $r_{tot} = 16800000 Symbols/s * 5bps = 84Mbps$.

III. The ARQ Petri Net

A modelling paradigm aimed at performance results from timed behaviour are stochastic Petri Nets (SPN) [8]. It has nice properties like structural analysis [9] and a state space which can be transformed into a Markov chain by means of computers. Every token is a quantum of information, e.g. a data packet or an OFDM symbol. Places model storage in buffers or the channel capacity. Transitions are serving elements, e.g.

[1] this definition is improved compared to the previous section.

a serial transmission, mainly "something happening". SPN are ideal for modeling timing behavior and circumstances like competition, choice, synchronization.

In Figure 6 a useful example of the SPN paradigm is shown. Here a Selective Repeat ARQ mechanism is modeled. The tokens represent packets. The channel is modeled by transitions. Packet errors are randomly introduced before the channel. Tokens move in closed circles so that the state space stays limited. An exception is the transition $unblock$. The load is modeled as $\rho = T_{server}/T_{source}$.

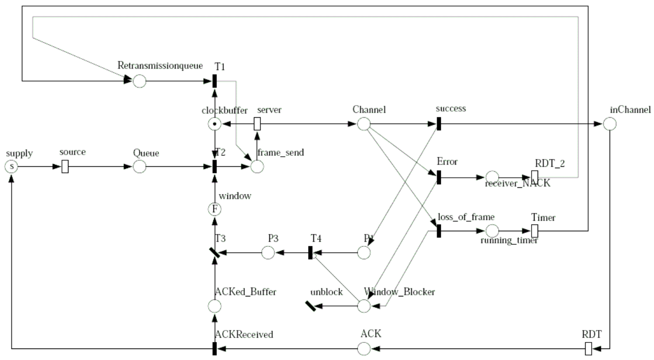

Fig. 6. Petri Net of a Selective Repeat ARQ Algorithm

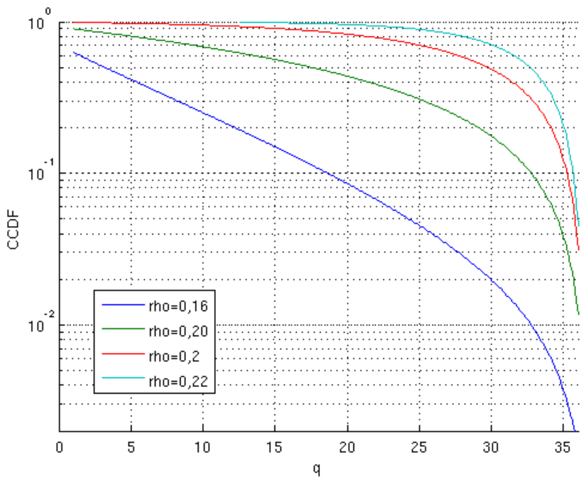

Fig. 7. CCDF of Queue Size for the Selective Repeat ARQ Algorithm ($window = 4$, $RTT = 4$, $Timer = 5$, $s = 40$, $P_{error} = 0.009$, $P_{loss} = 0.001$)

By using this model within the software package TimeNET [10] a Markov chain analysis can be performed to obtain numeric results for performance measures. All this can be performed numerically, without simulation. In this special model, the queue content distribution has been studied on the sender side, where the channel packet error probability was varied as a parameter. The result is visible in Figure 7. A variation of the error probability yields the results in Figure 8. Figure 9 shows the dependency of the average queue occupancy (limited buffer) from the offered load ρ and parameter f = window size.

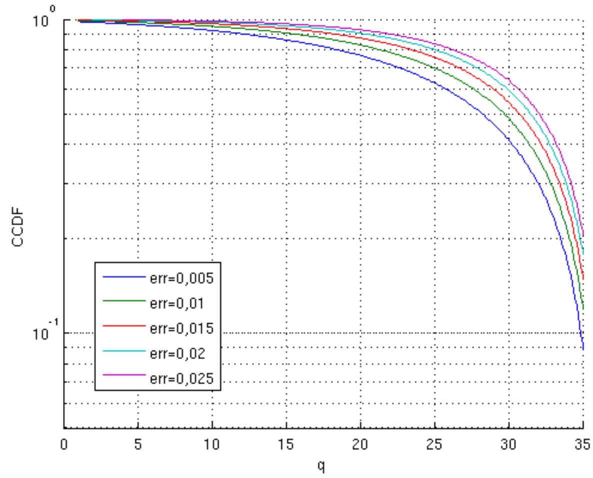

Fig. 8. CCDF of Queue Size for SR-ARQ ($window = 4$, $RTT = 4$, $\rho = 0.2$, $Timer = 5$, $s = 40$, $P_{loss} = 0$)

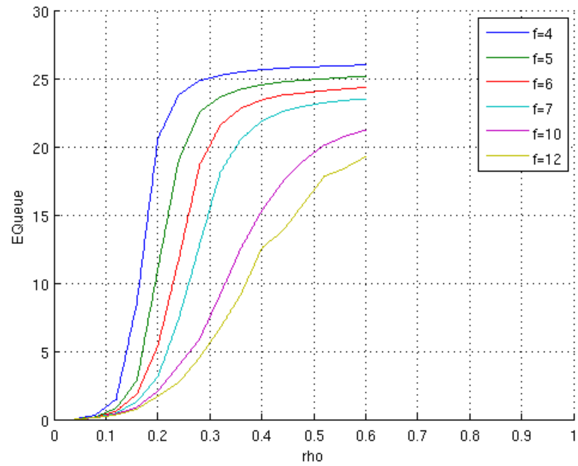

Fig. 9. $E(Queue)$ for SR-ARQ depending on ρ and parameter $f = window$ ($RTT = 4$, $Timer = 5$, $s = 30$, $P_{error} = 0.02$, $P_{loss} = 0.001$)

IV. Conclusion

This paper treats two unconventional models used in the area of wireless networking and OFDMA systems. An electric equivalent circuit was introduced to model the bitstream flows between a base station and user terminals. The model takes into account the different possible modulation and coding schemes due to the pathloss of terminals close and far from the base station. With this model the resource partitioning task between first and second hop could be solved.

A Petri Net (PN) model was also proposed to model the timing and performance behaviour of the ARQ retransmissions on layer 2 in a wireless system.

Results show that these paradigms are quite handy and easy to understand just by a view at its structure. So the benefit is a light bulb moment and a way shown to also use unconventional abstract models in fields where usually mathematical frameworks take much longer to understand and evaluate.

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