Unorthodox abstract models for the OFDMA multihop transmission
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Abstract—Wireless systems are complex devices and a lot of research is done to improve parts of these systems and get a better understanding of the total behaviour. One step towards an easier understanding is the ISO-OSI reference model which decomposes the system into layers. But still for each layer, easy models are missing where you can learn the behaviour within a few minutes. Researchers like to use abstract models in order to support an analytic approach and prefer detailed models that can only be evaluated by means of simulation. If this abstract view is desired, or a different view on a problem might be useful, an unconventional model can be handy. This paper introduces two models, an electric equivalent circuit and a Petri Net model, that were known for many decades from other technical areas and is used here to describe aspects of cellular radio systems, especially based on OFDM. They are not meant to be more accurate, but instead rather more to round off the high-level understanding.

I. INTRODUCTION

MODELLING of electrotechnical systems is always a tightrope walk between accuracy and clearness. Researchers like to use abstract models in order to support an analytic approach and prefer detailed models that can only be evaluated by means of simulation. In radio systems engineering, the mathematical models on the physical layer (PHY) are mature and complex models and formulas are very common. Sometimes it is desired to impart an overview knowledge about a topic or to get a different view on a problem from a standpoint that insiders cannot see anymore. That is when unconventional models come into place. They are not meant to be more accurate, but instead rather more to round off the high-level understanding.

In this paper an electric equivalent circuit is introduced which models the bit stream between base stations (BS) and user terminals (UT) as well as the required OFDMA resources depending on the possible PhyModes, which in turn depend on the pathloss, i.e. the distance between BS and UT. Especially when using multihop techniques, i.e. store-and-forward relays in cellular radio networks, the model can be used to learn the benefit of relaying. It can also be used for quantitative calculations, e.g. to determine the suitable resource partitioning scheme between first and second hop, or to find out the fractions of greedy (best effort) data streams that UTs will get if the resource scheduler assigns resources fairly among UTs.

These multihop radio systems are very beneficial due to their improvements of coverage and capacity. For that reason they are proposed for the next generation cellular systems like 3G-LTE [1]. The system capacity or spectral efficiency [bits/s/Hz] is improved without the need of more BSs with fiber access. In regular hexagonal scenarios [2] and city scenarios [3] this has been shown.

Fig. 1. Left: Conventional cellular geometry, Middle: Geometry to increase capacity, Right: Geometry to increase coverage

II. MODELLING OFDMA RESOURCES

OFDMA Resources on the radio link are the scarce resource that all UTs compete for when the offered traffic load is greedy, i.e. there is more traffic than capacity. This is the assumed situation for best effort traffic and typical file transfer or peer-to-peer applications.

A. OFDMA Resources

OFDM has evolved as an efficient multiplex scheme of typically 1024 small orthogonal subcarriers within the system bandwidth. The big advantage of OFDM is that different modulation and coding schemes (MCS, PhyMode) are possible for each subcarrier, so a robust BPSK can be used on frequencies where the channel is currently bad due to fading, while e.g.
QAM64 can be used on more stable subcarriers with higher SINR values. This is called dynamic adaptive modulation and coding (AMC) [4]. OFDMA means that within one OFDM symbol, several different UT receivers can be addressed. So for example, 256 subcarriers are used for each of four UTs that receive the symbol. Dynamic subcarrier assignment (DSA) selects the best resources for each UT based on channel state information (CSI). As the distance and pathloss may be very different among the UTs, it is likely that the PhyModes used for each UT are very different within the full OFDM symbol.

Close to the base station, the higher received SINR value allows the highest Modulation&Coding scheme (PhyMode 64QAM − 5/6 for LTE), which offers the highest raw data rate of approximately 84Mbit/s. At the cell border and in significant fractions of the cell area, the offered data rate is one order of magnitude lower. What makes this even worse is that some terminals operating at the lowest PhyMode occupy a ten times bigger part of the spectrum than the same number of terminals operating at the highest PhyMode. That means the average cell capacity is overproportionally determined by the maximum possible rate at the outer regions.

The organization of resources is in time and frequency, see Figure 2. Subcarriers are grouped together to form subchannels, within which all subcarriers are treated the same. For a broadband FDD LTE system there are 100 subchannels available. Within a radio cell the resources are unique and non-overlapping to avoid intra-cell interference completely. The BS coordinates the resources such that each link uses the maximum possible rate at the outer regions. Close to the base station, the higher received SINR value allows the highest Modulation&Coding scheme (PhyMode 64QAM − 5/6 for LTE), which offers the highest raw data rate of approximately 84Mbit/s. At the cell border and in significant fractions of the cell area, the offered data rate is one order of magnitude lower. What makes this even worse is that some terminals operating at the lowest PhyMode occupy a ten times bigger part of the spectrum than the same number of terminals operating at the highest PhyMode. That means the average cell capacity is overproportionally determined by the maximum possible rate at the outer regions.

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The equivalents are given in Table I. In this equivalent model, an electrical current has the meaning of a data stream, and the strength of the current means the data rate in [bits/s]. through the block diagram element. The current source to the left is the base station (BS) and its downlink (DL) traffic. At each junction the traffic splits, and Kirchoff’s Current Law ensures the consistency (no loss). In this model, a voltage means the amount of OFDMA resources available or used. A voltage source means a BS which has a certain number of OFDM subcarriers available, which gives a “voltage” in Symbols/s.

E.g. for 3GPP-LTE, this is 1200 · 7Symbols/s/500µs = 16800000Symbols/s raw (PHY) data rate. As it is the physical meaning of voltage, this is in a similar way the “potential” of the transmission, something proportional to the system bandwidth. According to Norton and Thevenin theorems, a voltage source and current source together with its internal resistance can always be transformed into each other. Therefore only a current source is drawn in Figure 3 instead of a voltage source. The resistors shown have a characteristic conductance G, which is proportional to the PhyMode capability, i.e. they can conduct a number of bits per symbol, e.g. 5 bits for QAM64 − 5/6.

This is exactly the mutual information (MI) that can be calculated for a given SINR and PhyMode [7]. Of course this G depends on the distance (due to pathloss and received SINR) at the UT. Ohm’s Law I = U · G can be satisfied with these equivalent definitions. All user terminals UTx are greedy here, i.e. they try to get as much data rate as possible. Let the distribution of resources be fair such that each UT gets an equal share of the OFDM resources. A user terminal

<table>
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<th>equivalent</th>
<th>eq. unit</th>
</tr>
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<td>BaseStation</td>
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**TABLE I**

**EQUIVALENT MEANING FOR THE ELECTRIC CIRCUIT MULTIHOP SYSTEM**
UTs attached to the base station then receives a datarate proportional to its conductance G by construction, because Kirchhoff’s Current and Voltage Laws hold.

If there is a relay in the circuit as shown on the right, there are resources needed on the first hop (U_{Hop1}), which are “wasted”, and the resources used on the second hop (U_{Hop2}) are shared while the datarate splits to one of the remote user terminals RUT. Despite the waste in the first hop, due to improved PhyModes (RUTs and RN are close together) there can be a higher datarate through RUT in the multihop case compared to a UT in a singlehop case where the Phymode is bad (low G).

Figure 4 shows an example for greedy traffic. Let \( G_{UT3} = G_{UT4} = 5 \text{Bits}/\text{Symbol} = 5 \text{bps} \) because they are close to the RN. Let \( G_{RUT, Hop1} \) also be \( 2 \cdot 5 \text{bps} \) because there is a high antenna gain between BS and RN and the factor 2 is needed for two supported RUTs. Then the parallel resistors \( G \) can be combined to give the RN. Let \( G_{RUT, Hop2} \) also be \( 10 \text{bps} \). The remaining two resistors in series are equivalent to one with \( G_R = 5 \text{bps} \). This means they compete with a singlehop UT of the same capability \( G_{UT} = 5 \text{bps} \). Both will get the same amount of resources altogether. For the but for two-hop transmissions, half of the resources are given to the first hop, half to the second hop. So \( U_{Hop2} = U_{Source}/2 \) and the resulting data rates \( I_{UTx} \) are half of that of a comparable singlehop UT. In this model, the currents \( I_{UTx} \) are proportional to the data rate the UTs would get, so they provide useful relative results under the condition that each flow must get an equal share of the resources. For absolute results, the “voltage” source must be scaled (divided) by the number of participants of the resources.

![Figure 4](image-url)  
**Fig. 4.** Left: Singlehop UTs; Right: 2-hop RUTs; The PhyModes are different.

For the singlehop UTs, according to Figure 3 the data rate they get is \( I_{UT1} = U_{Resources} \cdot G_{UT1} \) and \( I_{UT2} = U_{Resources} \cdot G_{UT2} \). For these four UTs, \( U_{Resources} \) is \( 16800000 \text{Symbols/s}/4 = 4200000 \text{Symbols/s} \). Let’s assume \( G_{UT1} = 5 \text{bps} (QAM64 - \frac{3}{2}) \) and \( G_{UT2} = 2/3 \text{bps} (QPSK - \frac{3}{2}) \). Then the data rates are \( I_{UT1} = 21 \text{Mbps} \), \( I_{UT2} = 2.8 \text{Mbps} \), \( I_{RUT3} = I_{RUT4} = 10.5 \text{Mbps} \) and the OFDMA channel is fully utilized with that. The data rates (as the current does) add up to a total of \( I_{tot} = 44.8 \text{Mbps} \).

In general the resource partitioning and data rates can be calculated like that. In the special case when all UTs have the same PhyMode and there are as many UTs ans RUTs, the suitable resulting resource partitioning is shown in Figure 5. It is interesting to note that the links BS \( \rightarrow \) UT, BS \( \rightarrow \) RN, RN \( \rightarrow \) RUT all get 1/3 of the total resources (the BS1 resource block is used for BS \( \rightarrow \) UT and BS \( \rightarrow \) RN).

Figure 5. Resource partitioning between BS and RN. Shown here is which BS or RN uses which resource subset.

### C. Fair Share Partitioning

We can now derive the calculations for fair share partitioning and scheduling with arbitrary PhyModes [6]. The “proportional fair share” means that any UT gets the same amount of resources \( R_i \) on its last hop. This amount of resources leads to a higher data rate \( r_i \) for those UTs \( i \) using a higher PhyMode. Let \( MCS_i \) be the PhyMode on the last hop for \( UT_i \) or \( RUT_i \), and \( MI_i \) be the mutual information in [bits/s/Hz] for this PhyMode [7]. \( MI_{RN} \) be the mutual information of hop1 between BS and RN of a multihop connection. \( MI_{max} \) is \( 5 \text{bits/s/Hz} \) for QAM64 - 5/6. The amount of resources needed is \( R_i = c \cdot \frac{r_i}{MI_i} \) with a constant \( c \) that is cancelled out in the equations, later. The resources are limited as

\[
R_{tot} = c \cdot \frac{r_{tot}}{MI_{max}} = \sum R_i = \end{align}

\[\text{(1)}\]

\[
c \sum_{i=1}^{\#UTs} r_{UT_i} \cdot \frac{1}{MI_{UT_i}} + c \sum_{i=1}^{\#RUTs} r_{RUT_i} \cdot \frac{1}{MI_{RUT_i}} + c \sum_{i=1}^{\#UTs} \frac{r_{RUT_i}}{MI_{RN}}.\]

Eq. [1] can be used to calculate the data rates of each UT and the required resources. If the fairness policy is to grant each UT the same amount of resources \( R_i \) and to each RUT twice this number (to support hop1 and hop2 and a fair competition between UTs and RUTs) then we get using \( c = \frac{R_{tot}}{\#UTs \cdot MI_{max}} \)

\[\text{(2)}\]

\[R_i = R_{tot} \cdot \frac{1}{\#UTs + 2 \cdot \#RUTs}.\]

\[\text{(3)}\]

\[r_{tot} \text{ is the total maximum data rate } r_{tot} = 16800000 \text{Symbols/s} \cdot 5 \text{bps} = 84 \text{Mbps}.\]

### III. THE ARQ PETRI NET

A modelling paradigm aimed at performance results from timed behaviour are stochastic Petri Nets (SPN) [8]. It has nice properties like structural analysis [9] and a state space which can be transformed into a Markov chain by means of computers. Every token is a quantum of information, e.g. a data packet or an OFDM symbol. Places model storage in buffers or the channel capacity. Transitions are serving elements, e.g.

\[\text{this definition is improved compared to the previous section.}\]
a serial transmission, mainly "something happening". SPN are ideal for modeling timing behavior and circumstances like competition, choice, synchronization.

In Figure 6 a useful example of the SPN paradigm is shown. Here a Selective Repeat ARQ mechanism is modeled. The tokens represent packets. The channel is modeled by transitions. Packet errors are randomly introduced before the channel. Tokens move in closed circles so that the state space stays limited. An exception is the transition unblock. The load is modeled as \( \rho = \frac{T_{server}}{T_{source}} \).

![Fig. 6. Petri Net of a Selective Repeat ARQ Algorithm](image)

By using this model within the software package TimeNET [10] a Markov chain analysis can be performed to obtain numeric results for performance measures. All this can be performed numerically, without simulation. In this special model, the queue content distribution has been studied on the sender side, where the channel packet error probability was varied as a parameter. The result is visible in Figure 7. A variation of the error probability yields the results in Figure 8. Figure 9 shows the dependency of the average queue occupancy (limited buffer) from the offered load \( \rho \) and parameter \( f = \text{window} \).

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![Fig. 7. CCDF of Queue Size for the Selective Repeat ARQ Algorithm](image)

Fig. 8. CCDF of Queue Size for SR-ARQ (window = 4, RTT = 4, \( \rho = 0.2 \), Timer = 5, \( s = 40 \), \( P_{\text{loss}} = 0 \)).

IV. CONCLUSION

This paper treats two unconventional models used in the area of wireless networking and OFDMA systems. An electric equivalent circuit was introduced to model the bitstream flows between a base station and user terminals. The model takes into account the different possible modulation and coding schemes due to the pathloss of terminals close and far from the base station. With this model the resource partitioning task between first and second hop could be solved.

A Petri Net (PN) model was also proposed to model the timing and performance behaviour of the ARQ retransmissions on layer 2 in a wireless system. Results show that these paradigms are quite handy and easy to understand just by a view at its structure. So the benefit is a light bulb moment and a way shown to also use unconventional abstract models in fields where usually mathematical frameworks take much longer to understand and evaluate.
REFERENCES


